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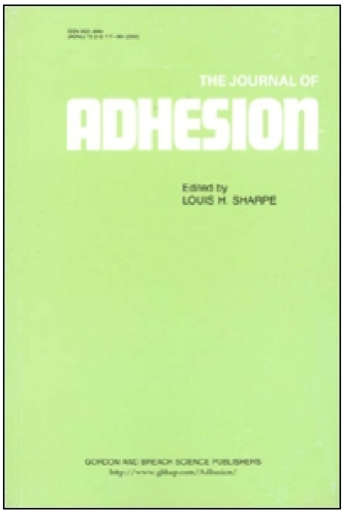

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Stability Analysis of a Nonlinear Blister Test

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The Griffith and stability criteria for the blister test have been derived from thermodynamic arguments. The Griffith criterion includes the stored energy in the blister as well as the interfacial energy change associated with debonding. Based on these criteria an analysis is given of a blister test with a neo-Hookian overlayer debonding from a rigid substrate. A closed analytic form for the strain energy release rate, *G*, is obtained valid for large elastic deformation of a thin circular film. The criteria derived indicate that quasi-static debonding of the blister is always stable, independent of the interfacial energies and the extent of blister deformation.

Keywords: Griffith and stability criteria; blister test; neo-Hookian; strain energy release rate

INTRODUCTION

The blister test has been developed to measure the adhesion strength between a deformable adhesive film and a rigid substrate. The test specimen typically consists of a thin flexible film layer on a perforated, rigid substrate. A fluid is injected at the film/substrate interface through the perforation, thereby causing a progressive debonding of the blistered overlayer film. The fracture energy can be obtained from the geometry and pressure of the blister.

Adhesion energy measurement by the blister test was first discussed by Dannenberg in 1961 [1]. Since then, it has been developed by

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Williams and co-workers [2–8] and other researchers [9–14], and modified for a variety of special applications [15–17]. Gent and Lewandowski [9] analyzed the blister test for a thin film on a rigid substrate, regarding the adhesive overlayer film as a linear elastic membrane and recognizing only the tensile rigidity of the film. According to their analysis, which uses an energy criterion for fracture in terms of a fixed characteristic work per unit area to separate the interface, two important relations are obtained. After the debonding begins, the product of the pressure, P, and the blister height, h, is proportional to the fracture energy, G_a, (considered a constant),

$$P \cdot h = 1.54\, G_a. \tag{1}$$

Secondly, the product of the pressure and the blister radius, a, is a constant including G_a,

$$P \cdot a = (17.4\, E d G_a^3)^{1/4} \tag{2}$$

where E and d are Young's modulus and thickness of the overlayer film, respectively.

On the basis of these two relationships, Chu and Durning [18] analyzed the blister test carried out at a constant volumetric flow rate of the injection fluid. A definite relationship for the time dependence of the debonding pressure, P, was obtained; P^{-3} vaires linearly with time, t, after debonding begins. With the slope of this linear relation, G_a can be calculated from

$$G_a = 0.39\,(R^2/N^2\, E d)^{1/5} \tag{3}$$

where R is the constant flow rate of fluid injection and N is the slope of the P^{-3} *versus* t relation. Since Eq. (3) is independent of the geometry of the blister which is usually difficult to measure, G_a can be simply determined by only measuring the pressure with time after debonding begins.

While the analyses mentioned were shown to describe well debonding behavior during the blister test in several systems, they do not say anything about the possibility of intermittent or unstable debonding processes. The criterion for the stability can be expressed in terms of quantities common in fracture mechanics such as the strain energy release rate, G, or the stress intensity factor, K [19–21]. The sign of the derivative of G or K with respect to debonded area or crack length determines the stability. This approach enables one to anticipate certain general aspects of debonding dynamics. Another limitation in prior analysis of the blister test is that all the previously-mentioned works are restricted to linear mechanics of the blister, the results of which can only be used for the cases of small deflections with a small debonding angle at the crack tip. The linear analysis cannot give an adequate understanding of the blister test on nonlinear materials such as elastomers or/and under large elastic deformations. Although few works have discussed nonlinear blister tests [23–25], none treat the stability of such a system.

In this work we analyze a nonlinear blister test, using thermodynamic and mechanical ideas. We first derive the Griffith and stability criteria for the blister test on the basis of thermodynamics, along the lines of the treatment of the "JKR" adhesion test discussed by Maugis [22]. We then derive the strain energy release rate, G, of a nonlinear elastic blister system. Based on the stability criterion and the G derived, we discuss the stability of a quasi-static nonlinear blister test.

THE GRIFFITH AND STABILITY CRITERIA FOR BLISTER TEST

Consider the system made of a thin film partially debonded from a rigid substrate and inflated by injected fluid (Fig. 1). The state of the system is defined by the blister volume, V, equal to the amount of fluid injected, the debonded area, A, allowed to vary independent of volume, and the temperature, T. Alternatively, we can replace T by the entropy, S. Then the total energy of the system, U, is a function of S, V, and A: $U = U(S, V, A)$, and the first differential of U can be written in the form,

$$dU = \left(\frac{\partial U}{\partial S}\right)_{V,A} dS + \left(\frac{\partial U}{\partial V}\right)_{S,A} dV + \left(\frac{\partial U}{\partial A}\right)_{S,V} dA \qquad (4)$$

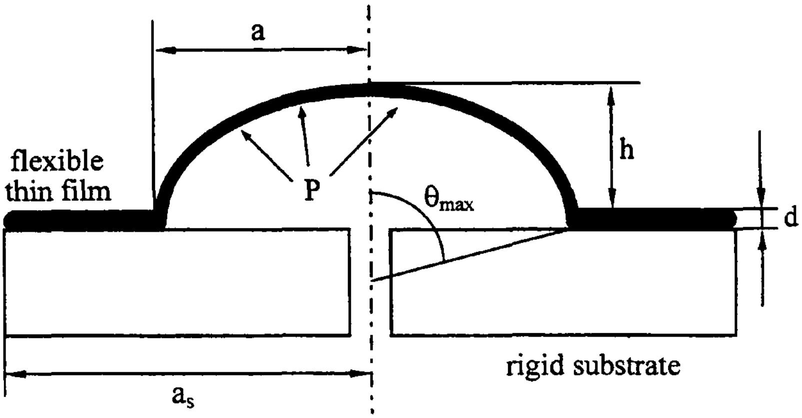

FIGURE 1 Inflation of circular thin film to spherical cap.

with

$$\left(\frac{\partial U}{\partial S}\right)_{V,A} = T, \quad \left(\frac{\partial U}{\partial V}\right)_{S,A} = -P, \quad \text{and}$$
$$\left(\frac{\partial U}{\partial A}\right)_{S,V} = \left(\frac{\partial U_E}{\partial A}\right)_{S,V} + \left(\frac{\partial U_S}{\partial A}\right)_{S,V}. \tag{5}$$

The first two are the usual source of heat and work for reversible changes. U_E is the stored elastic energy in the blister and U_S is the interfacial energy, expressed by

$$U_S = (A_0 - A)\gamma_{SF} + A\gamma_{SL} + A_B\gamma_{FL} \tag{6}$$

where γ_{SF}, γ_{SL}, and γ_{FL} are interfacial energies between substrate and overlayer film, substrate and liquid, and the film and liquid, respectively, and A_0 and A_B are the total substrate area and the internal surface area of the blister cap, respectively. Assuming a spherical cap, $A_B = \pi(a^2 + h^2)$ and $V = \pi(3a^2 + h^2)h/6$ where a and h are the radius and height of the blister, respectively. Actually, the volume of a blister is related to its radius and height through a volume parameter k: $V \cong ka^2h$. Since the variation in k during the propagation after debonding appears to be not large [12], we assume that k remains constant throughout debonding processes. Then, Eq. (6) is reduced to

$$U_S = w_{\mathrm{SFL}} A + \gamma_{\mathrm{SF}} A_0 + \gamma_{\mathrm{FL}} \pi^3 V^2/k^2 A^2 \tag{7}$$

where $w_{\mathrm{SFL}} = \gamma_{\mathrm{SL}} + \gamma_{\mathrm{FL}} - \gamma_{\mathrm{SF}}$. The strain energy release rate, G, is defined by

$$G = -\left(\frac{\partial U_E}{\partial A}\right)_{S,V}. \tag{8}$$

Therefore, Eq. (4) becomes

$$dU = TdS - PdV + (w_{\mathrm{SFL}} - 2\gamma_{\mathrm{FL}} \pi^3 V^2/k^2 A^3 - G)dA. \tag{9}$$

For a reversible and isothermal transformation, the free energy change of the system, F, is

$$dF = dU - d(TS) = -PdV + (w_{\mathrm{SFL}} - 2\gamma_{\mathrm{FL}} \pi^3 V^2/k^2 A^3 - G)dA. \tag{10}$$

From $(dF/dA)_V = 0$, equilibrium at constant volume is given by

$$G = w_{\mathrm{SFL}} - 2\gamma_{\mathrm{FL}} \pi^3 V^2/k^2 A^3 = w_{\mathrm{SFL}} - 2\gamma_{\mathrm{FL}} h^2/a^2. \tag{11}$$

This equilibrium relation is the Griffith criterion for the blister test. At equilibrium G corresponds to the "true" adhesion energy. Interestingly, the Griffith criterion is comprised of the blister geometry as well as the intrinsic interfacial energy, *i.e.*, the thermodynamic energy of adhesion. If $G \neq w_{\mathrm{SFL}} - 2\gamma_{\mathrm{FL}} \pi^3 V^2/k^2 A^3$, the debonded area will spontaneously change in order to decrease the thermodynamic potential. At constant volume, if $G < w_{\mathrm{SFL}} - 2\gamma_{\mathrm{FL}} \pi^3 V^2/k^2 A^3$, the debonded area must decrease to give $dF < 0$, *i.e.*, the blister radius reduces. Conversely, if $G > w_{\mathrm{SFL}} - 2\gamma_{\mathrm{FL}} \pi^3 V^2/k^2 A^3$, A must increase and thereby the blister increases.

The equilibrium given by Eq. (11) can be stable, unstable, or metastable. A thermodynamic system under a given constraint is stable if the corresponding thermodynamic potential is minimum, that is, if its second derivative is positive. Thus, the stability of the system is defined by

$$\left(\frac{\partial^2 F}{\partial A^2}\right)_V = F_{AA,V} = \frac{6\gamma_{\mathrm{FL}} \pi^3 V^2}{k^2 A^4} - \left(\frac{\partial G}{\partial A}\right)_V > 0 \tag{12}$$

at constant volume. $F_{AA,V}$ also contains the term related to the blister geometry.

STABILITY OF A NONLINEAR BLISTER TEST

The pressure inside a blister, P, can be expressed in terms of independent extensive parameters: $P = P(S, V, A)$. For large deflections, the nonlinearity of the system should be considered. Adopting a neo-Hookian constitutive relation for the nonlinearity, from the calculations on the large spherical inflation of a circular thin film by Adkins and Rivlin [26] and Hart-Smith and Crisp [27], the pressure, P, within a spherical blister (Fig. 1) is

$$P = 8C_1 d \frac{(\lambda - 1)^{1/2}}{a\lambda}\left[1 - \frac{1}{\lambda^6}\right] \tag{13}$$

where C_1 and d are a material constant and the thickness of the film before inflation, respectively. λ is the extension ratio of the film occurring at the blister pole, expressed as

$$\lambda = \frac{1}{\cos^2((1/2)\theta_{\max})} = \frac{a^2 + h^2}{a^2} \tag{14}$$

resulting in

$$P = 8C_1 d \frac{ka^2 V}{k^2 a^6 + V^2}\left[1 - \left(\frac{k^2 a^6}{k^2 a^6 + V^2}\right)^6\right]. \tag{15}$$

Assuming the blister being in equilibrium during debonding *i.e.*, a quasi-static debonding process, and a constant k, and considering a as an independent variable equivalent to A, the stored elastic energy during inflation is obtained as

$$U_E = \int_{a=\text{constant}} P dV = \int 8C_1\, d \frac{ka^2 V}{k^2 a^6 + V^2}\left[1 - \left(\frac{k^2 a^6}{k^2 a^6 + V^2}\right)^6\right] dV. \tag{16}$$

Letting $(k^2a^6)/(k^2a^6+V^2)=q$, we have

$$U_E=-4C_1dka^2\int_1^{(k^2a^6/k^2a^6+V^2)} q^{-1}(1-q^6)\,dq \tag{17}$$

$$=4C_1dka^2\left[\frac{1}{6}\left(\frac{k^2a^6}{k^2a^6+V^2}\right)^6-\ln\left(\frac{k^2a^6}{k^2a^6+V^2}\right)-\frac{1}{6}\right] \tag{18}$$

$$=\frac{4C_1dk}{\pi}A\left[\frac{1}{6}\left(\frac{k^2A^3}{k^2A^3+\pi^3V^2}\right)^6-\ln\left(\frac{k^2A^3}{k^2A^3+\pi^3V^2}\right)-\frac{1}{6}\right]. \tag{19}$$

From Eqs. (8) and (19), the strain energy release rate, G, and its first derivative with respect to debonded area at constant volume are derived as

$$\begin{aligned}G&=-\left(\frac{\partial U_E}{\partial A}\right)_V\\&=\frac{4C_1dk}{\pi}\left[\ln\left(\frac{k^2A^3}{k^2A^3+\pi^3V^2}\right)\right.\\&\quad\left.+\frac{1}{6}\left(\frac{k^2A^3+19\pi^3V^2}{k^2A^3+\pi^3V^2}\right)\left\{1-\left(\frac{k^2A^3}{k^2A^3+\pi^3V^2}\right)^6\right\}\right]\end{aligned} \tag{20}$$

and

$$\begin{aligned}\left(\frac{\partial G}{\partial A}\right)_V&=\frac{12C_1dk^3\pi^2A^2V^2}{(k^2A^3+\pi^3V^2)^2}\left[\frac{k^2A^3+\pi^3V^2}{k^2A^3}-3\right.\\&\quad\left.+\left(3-\frac{k^2A^3+19\pi^3V^2}{k^2A^3}\right)\left(\frac{k^2A^3}{k^2A^3+\pi^3V^2}\right)^6\right]\end{aligned} \tag{21}$$

respectively. If a and h are divided by the characteristic length, a_s (the dimension of the substrate), that is, $\bar{a}=a/a_s$ and $\bar{h}=(h/a_s)\cdot(\pi^{3/2}/k)$, then the dimensionless debonded area and volume are $\bar{A}=\pi\bar{a}^2=A/a_s^2$ and $\bar{V}=k\bar{a}^2\bar{h}=\pi^{3/2}V/(ka_s^3)$. Then, from Eqs. (11) and (20), the equilibrium condition of the nonlinear system in the dimensionless form is given, with $\bar{\gamma}_{\mathrm{FL}}=\gamma_{\mathrm{FL}}/(2C_1dk/\pi)$ and $\bar{w}_{\mathrm{SFL}}=$

$w_{\mathrm{SFL}}/(2C_1 dk/\pi)$, by

$$\frac{1}{2}\bar{w}_{\mathrm{SFL}} - \frac{\bar{V}^2}{\bar{A}^3}\cdot\bar{\gamma}_{\mathrm{FL}} = \ln\left(\frac{\bar{A}^3}{\bar{A}^3+\bar{V}^2}\right) + \frac{1}{6}\left(\frac{\bar{A}^3+19\bar{V}^2}{\bar{A}^3+\bar{V}^2}\right)\left\{1-\left(\frac{\bar{A}^3}{\bar{A}^3+\bar{V}^2}\right)^6\right\}. \tag{22}$$

The material constant C_1 is typically in the range of $1\times10^5 - 3\times10^5\,\mathrm{N/m^2}$ for rubbers [28] and k for a blister ranges from $\pi/2$ to $2\pi/3$ for $h \le a$. Assuming w_{SFL} and γ_{FL} in the range of 0–80 mN/m and d of 10–80 μm, $\bar{w}_{\mathrm{SFL}}$ and $\bar{\gamma}_{\mathrm{FL}}$ have values between 0 and 0.08. The numerical solution of $\bar{V}^2/\bar{A}^3$ satisfying Eq. (22) with various values of $\bar{w}_{\mathrm{SFL}}$ and $\bar{\gamma}_{\mathrm{FL}}$ is shown in Figure 2. $\bar{V}^2/\bar{A}^3(=h^2/a^2)$ increases with $\bar{w}_{\mathrm{SFL}}$ and is relatively insensitive to $\bar{\gamma}_{\mathrm{FL}}$. $\bar{w}_{\mathrm{SFL}}$ increases when w_{SFL} increases or C_1 and d decrease. This feature indicates that a more elastic material, a thinner film, or large intrinsic interfacial energy produces a bulgier blister, *i.e.*, large h/a.

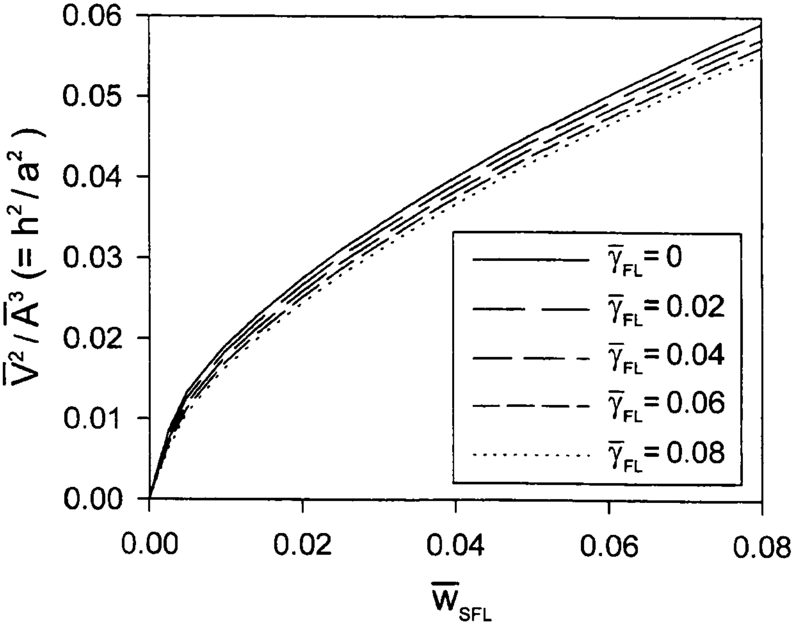

FIGURE 2 $\bar{V}^2/\bar{A}^3$ satisfying equilibrium condition for various values of $\bar{w}_{\mathrm{SFL}}$ and $\bar{\gamma}_{\mathrm{FL}}$.

Eqs. (12) and (21) lead to the dimensionless stability condition of the nonlinear system derived as

$$\begin{aligned}\bar F_{AA,V} &= \frac{\bar V^2}{\bar A^4}\left[\bar\gamma_{\mathrm{FL}} - \frac{\bar A^6}{(\bar A^3+\bar V^2)^2}\left\{\frac{\bar A^3+\bar V^2}{\bar A^3} - 3\right.\right.\\ &\qquad \left.\left. + \left(3-\frac{\bar A^3+19\bar V^2}{\bar A^3}\right)\left(\frac{\bar A^3}{\bar A^3+\bar V^2}\right)^6\right\}\right] \\ &= \frac{\bar V^2}{\bar A^4}\cdot \bar f_{AA,V} > 0\end{aligned} \tag{23}$$

where $\bar F_{AA,V} = F_{AA,V}/(12C_1dk/\pi a_s^2)$, *i.e.*,

$$\begin{aligned}\bar f_{AA,V} &= \bar\gamma_{\mathrm{FL}} - \frac{1}{(1+h^2/a^2)^2}\left[h^2/a^2 - 2 + (2-19h^2/a^2)\left(\frac{1}{1+h^2/a^2}\right)^6\right] \\ &= \bar\gamma_{\mathrm{FL}} - f(h^2/a^2) > 0\end{aligned} \tag{24}$$

For any value of $\bar V^2/\bar A^3 (= h^2/a^2)$ satisfying Eq. (22), $\bar f_{AA,V}$ is always positive regardless of $\bar w_{\mathrm{SFL}}$ and $\bar\gamma_{\mathrm{FL}}$ in the above range, as shown in Figure 3. This result is not surprising because $f(h^2/a^2)$ is negative for $0 < h^2/a^2 \leq 2$. as shown in Figure 4. Therefore, the nonlinear blister test system in quasi-static debonding processes is always stable at constant volume, and one should not see intermittent or unstable debonding processes.

There are few studies on nonlinear blister tests in the literature [23–25]; several analytic or numerical solutions for the energy release rate and debonding criteria have been discussed. Hohlfelder *et al.* [12] mentioned the possibility of unstable blister growth due to low stiffness of the test system. But none have analyzed the stability of debonding in a nonlinear blister system. The present approach, based upon assumed quasi-static debonding at constant volume, leads to a simple closed-form analysis of this problem. Further, the analytic solution can be applied to a linear elastic blister. Our calculation shows that debonding in a linear blister system, which corresponds to the case with small deflections, *i.e.*, $h^2/a^2 \ll 1$, is also stable under the same assumptions.

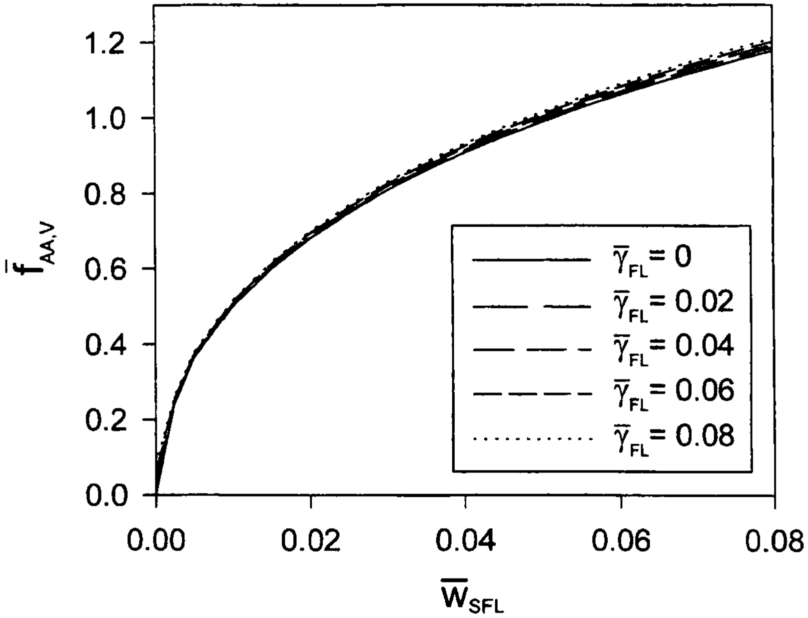

FIGURE 3 Plot of $\bar{f}_{AA,V}$ *versus* $\bar{w}_{SFL}$ for various values of $\bar{\gamma}_{FL}$.

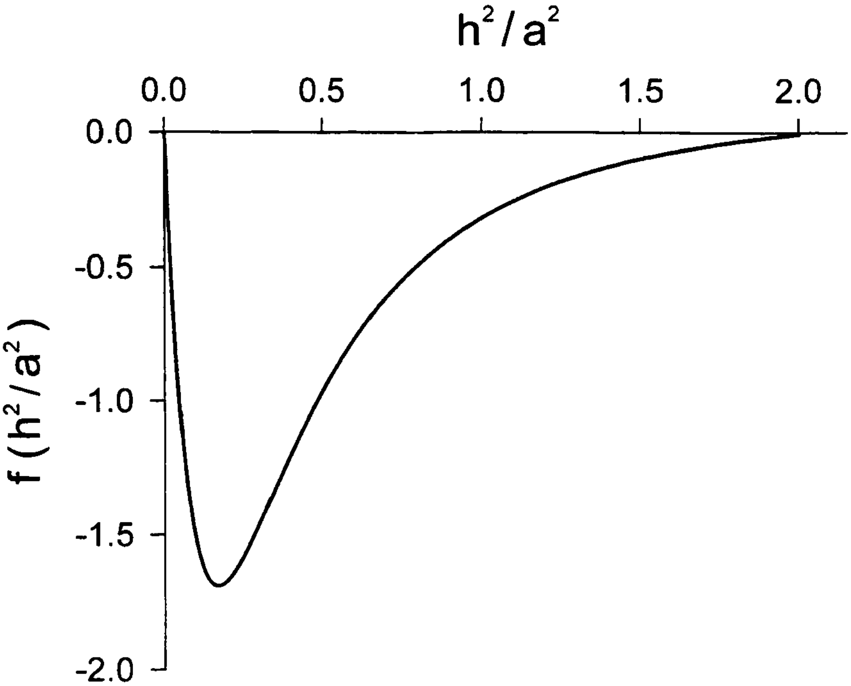

FIGURE 4 $f(h^2/a^2)$ *versus* h^2/a^2.

CONCLUSIONS

We derived the Griffith and stability criteria for the blister test on the basis of a thermodynamic approach. The criteria are functions of blister geometry as well as interfacial energies. An analysis of a nonlinear blister test is discussed connecting thermodynamic energy and fracture mechanics. For a nonlinear spherical blister of neo-Hookian material the strain energy release rate, G, is obtained valid for large elastic deformation of a thin circular film. The criteria derived indicate stability of quasi-static debonding of a nonlinear elastic film so that one should not see intermittent or unstable debonding processes in such a system.

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